Supplementary Methods

Numerical resolution of the Brownian equation

Justification of the damped limit

A general Langevin equation describing the Brownian evolution of a colloidal particle reads

$$\boldsymbol{m\ddot{x}(t) = -\xi|\dot{x}(t) - u_e(t)| - Kx(t) + \Gamma(t)} \qquad \textbf{(1)}$$

with in the present case ξ = 4,5.10^{-8} N.s.m^{-1} the Stokes friction coefficient of the bead, K = 1,0.10^{-4} N.m^{-1} the optical tweezers spring stiffness, u_e(t) the external pulsating flow of the order of a few mm.s^{-1}, m = 2,1.10^{-14} kg the mass of a bead of radius 1,5 μm and volume mass 1500 kg/m^3 (in absence of hydrodynamic corrections). Τ(t) is the random noise, or Langevin force. This random force is supposed to be a white, stationary and Gaussian noise, with the standard time correlations:

$$\boldsymbol{\langle\Gamma(t)\Gamma(t')\rangle = 2k_B T\xi\delta(t - t')} \qquad \textbf{(2)}$$

The overall process is Markovian.

Considering now the effect of periodic external force acting on the bead, one observes that three different regimes can be distinguished, delimited by two characteristic frequencies ω_s/(2π) and ω_i/(2π). For $\xi\omega_s$ = K, the friction force matches the tweezers restoring force, while for $\xi\omega_i = m\omega_i^2$ friction and inertia come even. Straightforward numerical estimates lead to ω_s/(2π)≈ 350 Hz and ω_i/(2π)≈340 kHz.

At low frequencies (f < 300 Hz), the position of the bead x(t) in the optical trap follows closely the external drag force ξu_e(t). At intermediate frequencies 300 < f < 3.10^5 Hz, the friction starts to cut the displacement off. At large frequencies, f > 3.10^5 Hz, inertia dominates and opposes even further the displacement. However, inertia can be safely neglected in the range of frequencies f > 10^4 Hz for which experimental sampling is done. Alternatively, one can say that the factor of quality of the mechanical device $Q = \sqrt{\omega_s/\omega_i} = \sqrt{Km/\xi} \simeq 0{,}03$ is low, and inertia play no role at low frequencies.

We therefore consider in this work the overdamped, or Smoluchowsky, limit of the Langevin equation :

$$-\xi\left(\dot{x}(t) - u_e(t) - Kx(t) + \Gamma(t)\right) = 0 \qquad (3)$$

Numerical resolution of the Langevin equation

One seeks to solve

$$\dot{x}(t) = u_e(t) - \frac{K}{\xi}x(t) + \frac{\Gamma(t)}{\xi} = 0 \qquad (4)$$

for any arbitrary flow u_e. One recognizes in the absence of flow a special instance of the exactly solvable Ornstein-Uhlenbeck process.

General algorithms for numerical solving stochastic differential equations can be found in specialized textbooks. Our Langevin equation is linear with additive noise and poses no special difficulty. The equation admits the following formal solution

$$x(t') = x't)e^{-\frac{K(t'-t)}{\xi}} + \int_t^{t'} ds.e^{-\frac{K(t'-s)}{\xi}} u_e(s) + \int_t^{t'} ds.e^{-\frac{K(t'-s)}{\xi}} \Gamma(s) \qquad (5)$$

The second term expresses the effect of the flow and justifies a numerical approach, given the non-trivial expression of $u_e(t)$. This formal solution is not restricted to small time intervals. Indeed, one is free to choose t and t' at will, and a natural choice is to integrate the equation between t and t + Δt with Δt the sampling time interval.

In practice, it is convenient to write the equation in terms of dimensionless variables. We picked up Tu = ξ/K = 4,5 .10^{-4} s as time unit, Vu≈u0 = 3,0 mm.s^{-1} as velocity unit and Lu = Vu.Tu = 1,35.10^{-6} m as length unit. In term of the dimensionless variables $\tilde{x} = \frac{x}{Lu}$, $\tilde{t} = t/Tu$, $\widetilde{u_e} = ue/Vu$, the equation becomes

$$d\tilde{x} = (\widetilde{u_e} - \tilde{x})d\tilde{t} + \sqrt{2\theta}d\widetilde{W} \qquad (6)$$

where $d\,\widetilde{W}$ is a normalized Wiener process, i.e. a Gaussian random variable of vanishing average and variance $d\tilde{t}$, while $\theta = \frac{kBT}{KLu^2}$ plays the role of dimensionless temperature scale. The discrete integration scheme now reads

$$\tilde{x}(\tilde{t} + \Delta\tilde{t}) = e^{-\Delta t}\tilde{x}(t) + \int_0^{\Delta\tilde{t}} d\tilde{s}\,.\,e^{-\tilde{s}}u_0(\tilde{t} + \Delta\tilde{t} - \widetilde{\tilde{s}) +} \sqrt{2\theta}\Delta\widetilde{W}(\tilde{t}) \qquad (7)$$

A discretisation time $\Delta\tilde{t} = 0{,}1\,(\Delta t = 4{,}5\,.\,10^{-5}\,s)$ is used in our simulations. The external flow has the following expression

$$\tilde{u}_e = 0.9\left|th\{3sin\,(\pi ft)\} - th\left\{3\,sin\left(\pi ft - {}^{\pi}/_{4}\right)\right\}exp\,(-0.79))\right| + 0.1 \qquad (8)$$

with f the heart beat frequency. Each $\boldsymbol{\Delta \widetilde{W}}\,(\boldsymbol{\tilde{t}})$ is an independent Gaussian random variable with zero average and variance $(1-e^{-2\Delta\tilde{t}})/2$. Such variables can be obtained starting from a uniform pseudo-random variable generator, and using the Box-Mueller algorithm, as described in Numerical recipe.

The integration of the $\tilde{u}_0$ term is done with the Gauss-Legendre sum on 8 points, a good compromise between speed and accuracy, which ensures validity so long as $\Delta\tilde{t}\ \sim 1$ and $\widetilde{u_0}$ stays smooth on such a length scale, which is the case in practice (the ow varies only on a typical 2 Hz frequency).

Spectral power of the random noise and the trajectories

To compare the computed trajectories to experimental power spectra obtained using a spectral analyzer, we perform fast Fourier transforms (FFT). Sample trajectories $x^{(d)}(ti)$ with i = 1…N = 2^p points (typically 2^{17} = 131072 points) are acquired and sine-transformed in order to approximate :

$$\tilde{x}^{(d)}(\omega_j) = \frac{\Delta t}{\sqrt{2\pi}}\sum_{k=0}^{N-1} exp\left(-i\frac{jk\pi}{N}\right)x^{(d)}(k\Delta t) \simeq \frac{1}{\sqrt{2\pi}}\int_0^{N\Delta t} dt.\,exp\,(-i\omega_j t)x(t) \qquad \textbf{(9)}$$

An approximation of the Fourier transformed continuous signal $\tilde{x}$ is therefore obtained on a discrete set of frequencies $\omega_j = j\Delta\omega$, equally spaced with interval $\Delta\omega = 2\pi/(N\Delta t)$. We repeat the cycle integration-FFT-integration-FFT . . . between 10 and 1000 times in order to average over the noise (here denoted $\langle.\rangle$).

The Fourier transform of the continuous noise Τ(t) is

$$\tilde{\Gamma}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt.\,e^{-i\omega t}\Gamma(t) \qquad \textit{(10)}$$

In the absence of external flow, one can write

$$\tilde{x}(\omega) = \frac{\tilde{\Gamma}(\omega)}{K + i\omega\xi}$$

$$\langle\tilde{x}(\omega)\widetilde{x^*}(\omega')\rangle = \frac{\langle\tilde{\Gamma}(\omega)\widetilde{\Gamma^*}(\omega\prime)\rangle}{(K+i\omega\zeta)(K+i\omega\zeta)*} \qquad \textbf{(11)}$$

From the definition of $\tilde{\Gamma}(\omega)$, one gets

$$\langle\tilde{\Gamma}(\omega)\widetilde{\Gamma^*}(\omega')\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty}dt\int_{-\infty}^{\infty}dt'\;2\xi k_B T\delta(t-t')e^{i\omega\prime t"-i\omega t}$$

$$= \frac{2\xi k_B T}{2\pi}\int_{-\infty}^{\infty}dt\,e^{i(\omega\prime-\omega)t}$$

$$= 2\xi k_B T \delta(\omega' - \omega) \quad (12)$$

In particular, the spectral power of $\tilde{x}$ over a narrow frequency interval $\Delta\omega$ reads

$$\langle \left| \int_{\omega}^{\omega+\Delta\omega} \tilde{x}(\omega') d\omega' \right|^2 \rangle \approx \frac{2k_B T\xi}{K^2 + \omega^2 \xi^2} \Delta\omega$$

The FFT of our discretized time trajectory approximates it in the following way

$$\Delta\omega^2 \langle \left| \sum_{\omega_1 < \omega_i < \omega_2} \widetilde{x^{(d)}}(\omega_j) \right|^2 \rangle \approx \langle \left| \int_{\omega_1}^{\omega_2} \tilde{x}(\omega') d\omega' \right|^2 \rangle$$

One can alteratively say that the average value $\langle \left| \widetilde{x^{(d)}}(\omega_j) \right|^2 \rangle$ is a numerical approximation of $(\Delta\omega)^{-2} \langle \left| \int_{\omega_i}^{\omega_i+\Delta\omega} \tilde{x}(\omega') d\omega' \right|^2 \rangle$.

For instance, in the case of a Gaussian white noise in real (SI) units one finds

$$\Delta\omega \langle \left| \widetilde{x^{(d)}}(\omega_j) \right|^2 \rangle \approx \frac{2k_B T\xi}{K^2 + \omega^2 \zeta^2}$$

We notice that the spectral density $2k_BT\xi/(K^2+\omega^2\zeta^2)$ has unit $m^2.s$, while $\left| x^{\widetilde{(d)}(\omega)} \right|^2$ is in $m^2.s^2$. A frequency interval $\Delta\omega$ is therefore needed to match both expressions.