Supplementary methods

Numerical resolution of the Brownian equation

Justification of the damped limit

A general Langevin equation describing the Brownian evolution of a colloidal particle reads

$$m\ddot{x}(t) = -\xi[\dot{x}(t) - u_e(t)] - Kx(t) + \Gamma(t) \quad (1)$$

with in the present case $\xi = 4.5 \times 10^{-8}$ N.s.m^{-1} the Stokes friction coefficient of the bead, $K = 1.0 \times 10^{-4}$ N.m^{-1} the optical tweezers spring stiffness, $u_e(t)$ the external pulsating flow of the order of a few mm.s^{-1}, $m = 2.1 \times 10^{-14}$ kg the mass of a bead of radius 1.5 μm and volume mass 1500 kg/m (in absence of hydrodynamic corrections).

$\Gamma(t)$ is the random noise, or Langevin force. This random force is supposed to be a white, stationary and Gaussian noise, with the standard time correlations [1] :

$$\langle \Gamma(t)\Gamma(t') \rangle = 2k_B T\xi\delta(t - t') \quad (2)$$

The overall process is Markovian.

Considering now the effect of periodic external force acting on the bead, one observes that three different regimes can be distinguished, delimited by two characteristic frequencies $\omega_s/(2\pi)$ and $\omega_i/(2\pi)$. For $\xi\omega_s = K$, the friction force matches the tweezers restoring force, while for $\xi\omega_i = m\omega_i^2$ friction and inertia come even. Straightforward numerical estimates lead to $\omega_s/(2\pi) \simeq$ 350 Hz and $\omega_i/(2\pi) \simeq 340$ kHz.

At low frequencies ($f < 300$ Hz), the position of the bead $x(t)$ in the optical trap follows closely the external drag force $\xi u_e(t)$. At intermediate frequencies $300 < f < 3 \times 10^5$ Hz, the friction starts to cut the displacement off. At large frequencies, $f > 3 \times 10^5$ Hz, inertia dominates and opposes even further the displacement. However, inertia can be safely neglected in the range of frequencies $f < 10^4$ Hz for which experimental sampling is done. Alternatively, one can say that the factor of quality of the mechanical device $Q = \sqrt{\omega_s/\omega_i} = \sqrt{Km}/\xi \simeq 0.03$ is low, and inertia play no role at low frequencies.

We therefore consider in this work the overdamped, or Smoluchowsky, limit of the Langevin equation :

$$-\xi[\dot{x}(t) - u_e(t)] - Kx(t) + \Gamma(t) = 0. \quad (3)$$

Numerical resolution of the Langevin equation

One seeks to solve

$$\dot{x}(t) = u_e(t) - \frac{K}{\xi}x(t) + \frac{\Gamma(t)}{\xi} = 0 \quad (4)$$

for any arbitrary flow u_e. One recognizes in the absence of flow a special instance of the exactly solvable Ornstein-Uhlenbeck process [1].

General algorithms for numerically solving stochastic differential equations can be found in specialized textbooks, *e.g.* [2]. Our Langevin equation is linear with additive noise and pose no special difficulty. The equation admits the following formal solution

$$x(t') = x(t)e^{-K(t'-t)/\xi} + \int_t^{t'} \mathrm{d}s\; e^{-K(t'-s)/\xi}u_e(s) + \int_t^{t'} \mathrm{d}s\; e^{-K(t'-s)/\xi}\Gamma(s) \quad (5)$$

The second term expresses the effect of the flow and justifies a numerical approach, given the non-trivial expression of $u_e(t)$. This formal solution is not restricted to small time intervals. Indeed, one is free to choose t and t' at will, and a natural choice is to integrate the equation between t and $t + \Delta t$ with Δt the sampling time interval.

In practice, it is convenient to write the equation in terms of dimensionless variables. We picked up $\mathrm{Tu} = \xi/K = 4.5 \times 10^{-4}$ s as time unit, $\mathrm{Vu} \simeq u_0 = 3.0$ mm.s^{-1} as velocity unit and $\mathrm{Lu} = \mathrm{Vu} \times \mathrm{Lu} = 1.35 \times 10^{-6}$ m as length unit. In term of the dimensionless variables $\tilde{x} = x/\mathrm{Lu}$, $\tilde{t} = t/\mathrm{Tu}$, $\tilde{u}_e = u_e/\mathrm{Vu}$, the equation becomes

$$\mathrm{d}\tilde{x} = (\tilde{u}_e - \tilde{x})\mathrm{d}\tilde{t} + \sqrt{2\theta}\mathrm{d}\tilde{W} \tag{6}$$

where $\mathrm{d}\tilde{W}$ is a normalized Wiener process, *i.e.* a Gaussian random variable of vanishing average and variance $\mathrm{d}\tilde{t}$, while $\theta = \frac{k_B T}{K \mathrm{Lu}^2}$ plays the role of dimensionless temperature scale. The discrete integration scheme now reads

$$\tilde{x}(\tilde{t} + \Delta\tilde{t}) = e^{-\Delta t}\tilde{x}(t) + \int_0^{\Delta\tilde{t}} \mathrm{d}\tilde{s}\, e^{-\tilde{s}}\tilde{u}_e(\tilde{t} + \Delta\tilde{t} - \tilde{s}) + \sqrt{2\theta}\Delta\tilde{W}(\tilde{t}) \tag{7}$$

A discretisation time $\Delta\tilde{t} = 0.1$ ($\Delta t = 4.5 \times 10^{-5}$ s) is used in our simulations. The external flow involves an hyperbolic tangent function, and has the following expression

$$\tilde{u}_e = \frac{u_e}{u_0} = 0.9|\tanh\{3\sin(\pi f t)\} - \tanh\{3\sin(\pi f t - \pi/4)\}\exp(-0.79))| + 0.1. \tag{8}$$

with f the heart beat frequency. Each $\Delta\tilde{W}(\tilde{t})$ is an independent Gaussian random variable with zero average and variance $(1 - e^{-2\Delta\tilde{t}})/2$. Such variables can be obtained starting from a uniform pseudo-random variable generator, and using the Box-Mueller algorithm, as described in [3].

The integration of the $\tilde{u}_e$ term is done with a Gauss-Legendre sum on 8 points, a good compromise between speed and accuracy, which ensures validity so long as $\Delta\tilde{t} \sim 1$ and $\tilde{u}_0$ stays smooth on such a length scale, which is the case in practice (the flow varies only on a typical 2 Hz frequency).

Spectral power of the random noise and the trajectories

To compare the computed trajectories to experimental power spectra obtained using a spectral analyzer, we perform fast Fourier transforms (FFT). Sample trajectories $x^{(d)}(t_i)$ with $i = 1 \dots N = 2^p$ points (typically $2^{17} = 131072$ points) are acquired and sine-transformed in order to approximate :

$$\hat{x}^{(d)}(\omega_j) = \frac{\Delta t}{\sqrt{2\pi}} \sum_{k=0}^{N-1} \exp\left(-i\frac{jk\pi}{N}\right) x^{(d)}(k\Delta t) \simeq \frac{1}{\sqrt{2\pi}} \int_0^{N\Delta t} \mathrm{d}t \exp(-i\omega_j t)x(t) \tag{9}$$

An approximation of the Fourier transformed continuous signal $\hat{x}$ is therefore obtained on a discrete set of frequencies $\omega_j = j\Delta\omega$, equally spaced with interval $\Delta\omega = 2\pi/(N\Delta t)$. We repeat the cycle integration-FFT-integration-FFT ... between 10 and 1000 times in order to average over the noise (here denoted $\langle . \rangle$).

The Fourier transform of the continuous noise $\Gamma(t)$ is

$$\hat{\Gamma}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}t\, e^{-i\omega t}\Gamma(t) \tag{10}$$

In the absence of external flow, one can write

$$
\begin{aligned}
\hat{x}(\omega) &= \frac{\hat{\Gamma}(\omega)}{K + i\omega\xi} \\
\left\langle \hat{x}(\omega)\hat{x}^*(\omega') \right\rangle &= \frac{\left\langle \hat{\Gamma}(\omega)\hat{\Gamma}^*(\omega') \right\rangle}{(K + i\omega\xi)(K + i\omega\xi)^*}
\end{aligned}
\tag{11}
$$

From the definition of $\hat{\Gamma}(\omega)$, one gets

$$
\begin{aligned}
\left\langle \hat{\Gamma}(\omega)\hat{\Gamma}^*(\omega') \right\rangle &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t \int_{-\infty}^{\infty} \mathrm{d}t' \, 2\xi k_b T \delta(t - t') e^{i\omega' t' - i\omega t} \\
&= \frac{2\xi k_b T}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t \, e^{i(\omega' - \omega)t} \\
&= 2\xi k_b T \delta(\omega' - \omega)
\end{aligned}
\tag{12}
$$

In particular, the spectral power of $\hat{x}$ over a narrow frequency interval $[\omega_1, \omega_2]$ reads

$$
\left\langle \left| \int_{\omega_1}^{\omega_2} \hat{x}(\omega') \mathrm{d}\omega' \right|^2 \right\rangle \simeq \frac{2k_B T\xi}{K^2 + \omega_1^2 \xi^2} (\omega_2 - \omega_1) \tag{13}
$$

The FFT of our discretised time trajectory, with $\Delta\omega$ the sampling frequency interval, approximates it in the following way

$$
\Delta\omega^2 \left\langle \left| \sum_{\omega_1 < \omega_i < \omega_2} \hat{x}^{(d)}(\omega_j) \right|^2 \right\rangle \simeq \left\langle \left| \int_{\omega_1}^{\omega_2} \hat{x}(\omega') \mathrm{d}\omega' \right|^2 \right\rangle \tag{14}
$$

One can alternatively say that the average value $\left\langle |\hat{x}^{(d)}(\omega_j)|^2 \right\rangle$ is a numerical approximation of $(\Delta\omega)^{-2} \left\langle \left| \int_{\omega_j}^{\omega_j + \Delta\omega} \hat{x}(\omega') \mathrm{d}\omega' \right|^2 \right\rangle$.

For instance, in the case of a Gaussian white noise in real (SI) units one finds

$$
\Delta\omega \left\langle \left| \hat{x}^{(d)}(\omega_j) \right|^2 \right\rangle \simeq \frac{2k_B T\xi}{K^2 + \omega_j^2 \xi^2} \tag{15}
$$

We notice that the spectral density $2k_B T\xi/(K^2 + \omega^2\xi^2)$ has unit $\mathrm{m}^2.\mathrm{s}$, while $|\hat{x}^{(d)}(\omega)|^2$ is in $\mathrm{m}^2.\mathrm{s}^2$. A frequency interval $\Delta\omega$ is therefore needed to match both expressions. The combination appearing in the left hand side of the above equation is invariant with respect to changes in the sampling frequency, and is used in the presence of an external flow as well.

Références

[1] C.W. Gardiner. *Handbook of Stochastic Methods for Physics,Chemistry and the Natural Sciences*. Springer-Verlag, 1985.

[2] P.E. Kloeden and E. Platen. *The Numerical Solution of Stochastic Differential Equations*. Springer-Verlag, 1999.

[3] William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery. *Numerical recipes in C*. Cambridge University Press, 1997.

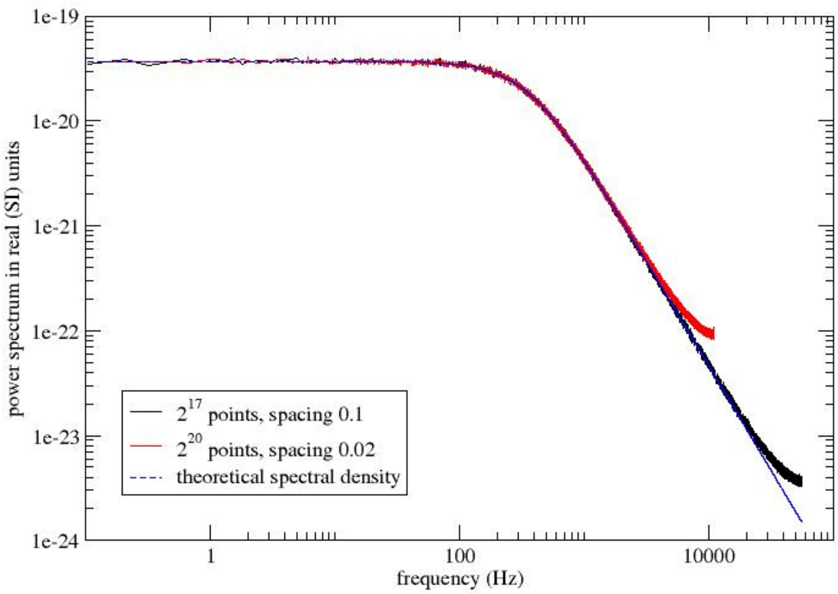

FIGURE 1 – Power spectra $\Delta\omega \left\langle |\hat{x}^{(d)}|^2 \right\rangle$ of the tweezers *vs* frequency $\omega/(2\pi)$ in the case of a pure white noise, and comparison with the theoretical spectrum, as given in eq. (15). Continuous red curve (stopping at $f = 10000$Hz) : 2^{17} points, $\Delta\tilde{t} = 0.1$, continuous black curve (stopping at $f = 50000$Hz) : 2^{20} points, $\Delta\tilde{t} = 0.02$. The agreement is extremely good, up to the highest frequencies where discretisation effects become visible, due to aliasing (high frequencies contributions of the noise folded back to the low frequency spectrum.)